Elliptic integrals

The complete elliptic integrals of the first and second kinds are defined as

$$K(k^2) = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1 - k^2 \sin^2 \theta}} \, d\theta \tag{1}$$

and

$$E(k^2) = \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 \sin^2 \theta} \, d\theta \tag{2}$$

respectively. They have the following properties:

$$K(0) = \frac{\pi}{2} \tag{3}$$

$$E(0) = \frac{\pi}{2} \tag{4}$$

$$K(1) = \tilde{\infty} \tag{5}$$

$$E(1) = 1 \tag{6}$$

$$\frac{d}{dk} K(k^2) = -\frac{K(k^2) + \frac{E(k^2)}{k^2 - 1}}{k} \tag{7}$$

$$\frac{d}{dk} E(k^2) = \frac{E(k^2) - K(k^2)}{k} \tag{8}$$

They also satisfy the Legendre relation:

$$E(k^2)K(1 - k^2) + E(1 - k^2)K(k^2) - K(k^2)K(1 - k^2) = \frac{\pi}{2} \tag{9}$$